Photonic Time-Crystals and Momentum Band-Gaps

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Abstract: Temporally periodic photonic-crystals develop an *ω-k* dispersion relation with momentum band-gaps, and offer new physical effects including frequency conversion, time-reversal and other optical phenomena in addition to their temporal control.
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1. Introduction

Photonic crystals consist of spatially varying structures that are periodic on the order of the wavelength of the incident light. They exhibit an ω-k dispersion relation with forbidden frequency zones commonly used in spontaneous emission inhibition [1], photon localization [2], high Q cavities, and wave-guiding. In wave equations, space and time play similar roles, allowing for temporally periodic materials, as shown in figure 2 (a), to provide optical properties complementary to the effects demonstrated by spatially periodic materials. Inspired by a space-time duality, we previously studied the temporal analog of metasurfaces [3], which were shown to offer new functionality compared to those offered by space-gradient metasurfaces alone [4]. This has inspired us to consider expanding the work to include photonic-crystals, metamaterials, and metasurface devices which currently control light through *spatial engineering* of optical properties. However, all of these devices can be further advanced, through *temporal engineering* of optical properties allowing for a new breed of optical devices and novel physical effects to be realized. In this work, we study photonic time-crystals demonstrating ω-k dispersion relations with forbidden momentum zones, and concluded by discussing possible applications

2. Theory and Numerical Verifications

In order to understand the optical properties demonstrated by photonic time-crystals a material whose permittivity changes periodically in time as is shown in figure 2(a), we start with the study of a single temporal boundary. We define this temporal boundary as the time at which the crystal material changes instantaneously from $\varepsilon_1(n_1)$ to $\varepsilon_2(n_2)$. In literature this phenomenon is called time-refraction [5-7]. Time refraction is the result of the time continuity of EM waves which ensures the wave momentum $k=n\omega/c$ will be conserved regardless of the time variation of n. This results in a frequency conversion of the incident light from to $\omega_2=n_1\omega_1/n_2$[7]. Additionally, another time-reversed wave is generated to satisfy continuity for both electric and magnetic fields. Figures 1(a-b) demonstrate the FDTD simulation that verifies the frequency conversion effects. Figure 1(a) demonstrates the spatiotemporal development of an optical pulse propagating through a finite thickness 1D time varying medium which is undergoing a refractive index shift from n_1=1.5 to n_2=3. As a result of this index change, the frequency of the incident pulse is down-converted to half its value (free-space wavelength is doubled) as is indicated from the FFT of the output pulses as shown in figure 1(b).

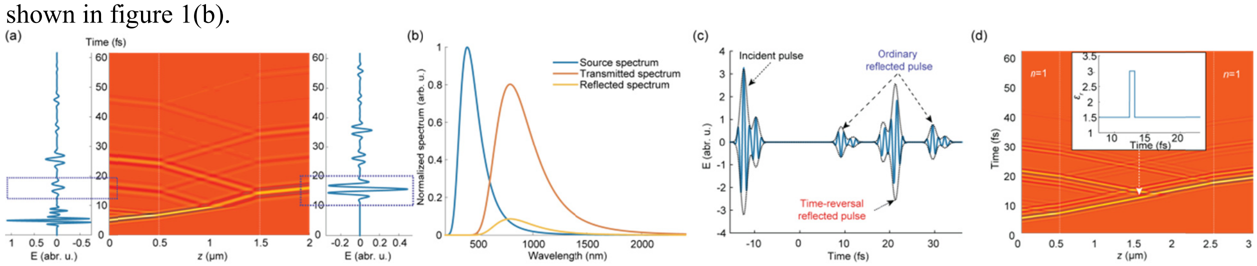

Figure 1. (a-b) FDTD simulation of a pulse going through a temporal boundary introduced inside a 1D slab. (d) FFT of transmitted and reflected pulses from the time-boundary demonstrating the frequency conversion effect. (c-d) Demonstration of time-reversal from a slab undergoing a pulsed change in its optical properties.

Figure 1(c-d) represents the FDTD simulation demonstrating the time-reversal effect where a non-symmetric incident pulse is applied to the medium. Subsequently, a time-reversal of the pulse envelope is observed in reflection.

To develop the ω-k relation of the structure in figure 2(a) which has a period $T=t_1+t_2$ where $t_1(t_2)$ is the time period over which the medium carries a value of $n_1(n_2)$. Transfer matrix analysis across the temporal layers along with a temporal analog of the Floquet condition ($f(t)=e^{-i\omega T}\ u(t)$; where $u(t+T)=\ u(t)$) leads to the following dispersion relation:

$$\cos\left(\omega T\right)=\cos\left(\omega_1 t_1\right)\cos\left(\omega_2 t_2\right)-\frac{{n_1}^2+{n_2}^2}{2n_1 n_2}\sin\left(\omega_1 t_1\right)\sin\left(\omega_2 t_2\right) \qquad (1)$$

where $\omega_1=ck/n_1$ and $\omega_2=ck/n_2$. Figure 2(b) shows the plot of the ω-k dispersion relation given by equation (1) for n_1=1.5 and n_2=3, indicating bandgaps along the k axis, in contrast with traditional photonic crystals where the bandgaps are for values of ω. In many physical processes such as optical nonlinearities and spontaneous emission, the role of k is as important as ω, since k controls momentum conservation and ω energy conservation (e.g, phase matching in optical non-linearities is controlled by k values). Figure 2(c-d) shows numerical verification of the band-gap effect where a pulse is propagating inside the time-varying medium with a momentum inside the forbidden region. The spectrum lying in the forbidden band is attenuated after undergoing a few cyclic changes to the index of the medium.

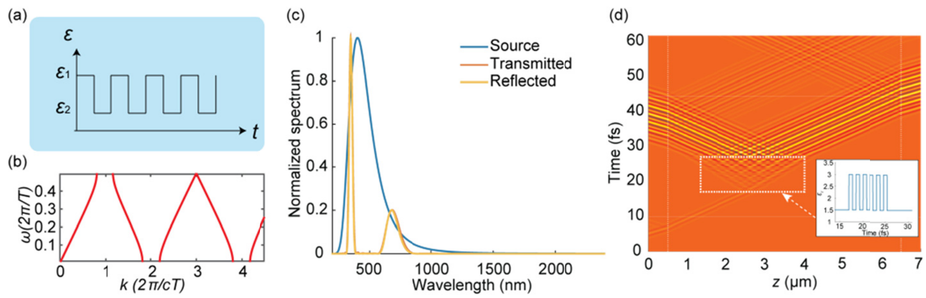

Figure 2. (a) Schematic of a photonic time-crystal with periodic change in optical properties. (b) Dispersion relation of photonic time-crystal with forbidden momentum zones. (c-d) Demonstration of band-gap effect of photonic time-crystals, where the spectrum of an optical pulse is attenuated within the band-gaps while propagating through a photonic time-crystal.

3. Conclusion

Photonic time-crystals have been introduced which exhibit photonic properties complementary to their conventional spatial photonic crystal analogs. While spatial photonic crystals induce forbidden bands in the frequency (energy) spectrum of photons, photonic time-crystals create forbidden regions in the wavenumber (momentum) spectrum of photons. This effect allows for enhanced control over many optical processes that require both photonic energy and momentum conservations such as nonlinear harmonic generation. In addition, the synchronization of spontaneous emission can be achieved through temporal tunability of forbidden regions. Moreover, the effects of spatial photonic crystal are only obtained in structures consisting of many spatial periods. On the other hand, there is no fundamental limitation on the minimum size of photonic time-crystals required to observe these effects.

4. Acknowledgement

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5. References

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